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Radiative Polarization in High-Energy Storage Rings *

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RADIATIVE POLARIZATION IN HIGH-ENERGY STORAGE RINGS

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Abstract

Electron and positron beams circulating in high-energy storage rings become spontaneously polarized by the emission of synchrotron radiation. The asymptotic degree of polarization that can be attained is strongly affected by so-called depolarizing resonances. Detailed experimental measurements of the polarization were made SPEAR about ten years ago, but due to lack of a suitable theory only a limited theoretical fit to the data has so far been achieved. I present a general formalism for calculating depolarizing resonances, which has been coded into a computer program called SMILE, and use it to fit the SPEAR data. By the use of suitable approximations, I am able to fit both higher order and nonlinear resonances, and thereby to interpret many hitherto unexplained features in the data, and to resolve a puzzle concerning the asymmetry of certain resonance widths seen in the data.

Introduction

It was predicted by Sokolov and Ternov [1] that electrons and positrons circulating in high energy storage rings would become polarized by the emission of synchrotron radiation; this effect is now called the "Sokolov-Ternov effect." They solved the Dirac equation in a homogenous vertical magnetic field and predicted an asymptotic degree of polarization of $8/(5\sqrt{3}) \simeq 92.4\%$. In practice, the polarization is sometimes reduced from this value by so-called "depolarizing spin resonances." A formula for the polarization including these resonances was derived by Derbenev and Kondratenko, [2] and is now known as the Derbenev-Kondratenko formula. Experimental measurements of the polarization, showing several resonances, were made at the storage ring SPEAR [3]. Only the first order resonances in these data have been fitted; [4] the widths of the rest have remained unexplained up to now. A formalism to calculate the widths of arbitrary spin resonances was given in Ref. [5], and a computer program called SMILE was written based on it. This program has been used to fit the SPEAR data, after making various approximations, and I have used it to explain various puzzling features seen in the data. Some of this work was presented in Ref. [6], and more details in Ref. [7], including new theoretical predictions which could serve as a check on the theory.

General Remarks

The Derbenev-Kondratenko formula is [2]

$$P_{eq} = \frac{8}{5\sqrt{3}} \frac{\left\langle \frac{1}{|\rho|^3}\hat{b}.\left(\hat{n} - \gamma\frac{\partial\hat{n}}{\partial\gamma}\right)\right\rangle}{\left\langle \frac{1}{|\rho|^3}\left(1 - \frac{2}{9}(\hat{n}.\hat{v})^2 + \frac{11}{18}\left|\gamma\frac{\partial\hat{n}}{\partial\gamma}\right|^2\right)\right\rangle}, \tag{1}$$

where P_{eq} is the equilibrium degree of polarization, ρ is the local radius of curvature of the particle trajectory, $\hat{v}$ is the direction of particle motion, $\hat{b} = \vec{v} \times \dot{\vec{v}}/|\vec{v} \times \dot{\vec{v}}|$, $\hat{n}$ is the spin quantization axis on a trajectory, and the angular brackets denote an ensemble average over the particle trajectories and accelerator azimuth. This formula was rederived, and the notation clarified, in Ref. [8]. The principal details are given in Ref. [9]. The algorithm in Ref. [5] evaluates the vectors $\hat{n}$ and $\gamma(\partial\hat{n}/\partial\gamma)$ using a perturbation expansion, to be described briefly below. The various spin resonances are obtained by systematically expanding the perturbation series. A copy of the data in Ref. [3] is shown in the inset to Fig. 1. Note that the curve therin is a guide to the eye, not a theoretical fit.

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Calculation of Resonances

A brief description of the algorithm in Ref. [5], frequently called the "SMILE algorithm," will be given below. It treats all resonances, in principle, in the approximation of linear orbital dynamics. A more general algorithm which also treats nonlinear orbital dynamics has been published by Yokoya [10]. A number of more limited algorithms, which treat a subset of the resonances included in the above algorithms, are given in Refs. [11] – [15]. I have proved the algorithms in Refs. [5,11,12,13,15] to be mathematically equivalent, when restricted to a common domain of approximation, in either Ref. [5] or [6]. Ref. [14] has been shown to be equivalent to the rest in Ref. [17]. The above are all analytical algorithms. A numerical tracking algorithm has been presented in Ref. [16], which also treats arbitrary resonances.

I now restrict attention to the SMILE algorithm [5]. I denote the accelerator azimuth by θ. The equation of motion for $\hat{n}$ is

$$\frac{d\hat{n}}{d\theta} = \vec{\Omega} \times \hat{n}, \tag{2}$$

where $\vec{\Omega}$ is the spin precession vector of the storage ring [18]. I decompose $\vec{\Omega} = \vec{\Omega}_0 + \vec{\omega}$, where $\vec{\Omega}_0$ is the value of $\vec{\Omega}$ on the accelerator closed orbit, and $\vec{\omega}$ describes the additional terms due to orbital oscillations around the closed orbit. Let $\hat{n}_0$ denote the value of $\hat{n}$ on the closed orbit, and let $\hat{l}_0$ and $\hat{m}_0$ denote the two other linearly independent solutions of Eq. 2 on the closed orbit: $d\hat{l}_0/d\theta = \vec{\Omega}_0 \times \hat{l}_0$, etc. Numerical solution of Eq. 2 for $\{\hat{l}_0, \hat{m}_0, \hat{n}_0\}$ is by now standard.

I now write $\hat{n} = n_1\hat{l}_0 + n_2\hat{m}_0 + n_3\hat{n}_0$, and define spherical harmonics via $V_{\pm 1} = \mp(n_1 \pm i n_2)/\sqrt{2}$, $V_0 = n_3$. The equation of motion for $V_{\pm 1}$ and V_0 is

$$\frac{d}{d\theta}\begin{pmatrix} V_1 \\ V_0 \\ V_{-1} \end{pmatrix} = i\vec{\omega}.\vec{J}^T \begin{pmatrix} V_1 \\ V_0 \\ V_{-1} \end{pmatrix}. \tag{3}$$

The formal solution is [5]

$$\begin{pmatrix} V_1 \\ V_0 \\ V_{-1} \end{pmatrix} = \mathrm{T}\left\{\exp\left(\int_{-\infty}^{\theta} i\vec{\omega}.\vec{J}^T\, d\theta'\right)\right\}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \tag{4}$$

The symbol "T" denotes a time-ordered, or θ-ordered, product. To obtain a practical solution, I expand the exponential in a power series, and evaluate the resulting integrals term by term: this is the SMILE perturbation series [5].

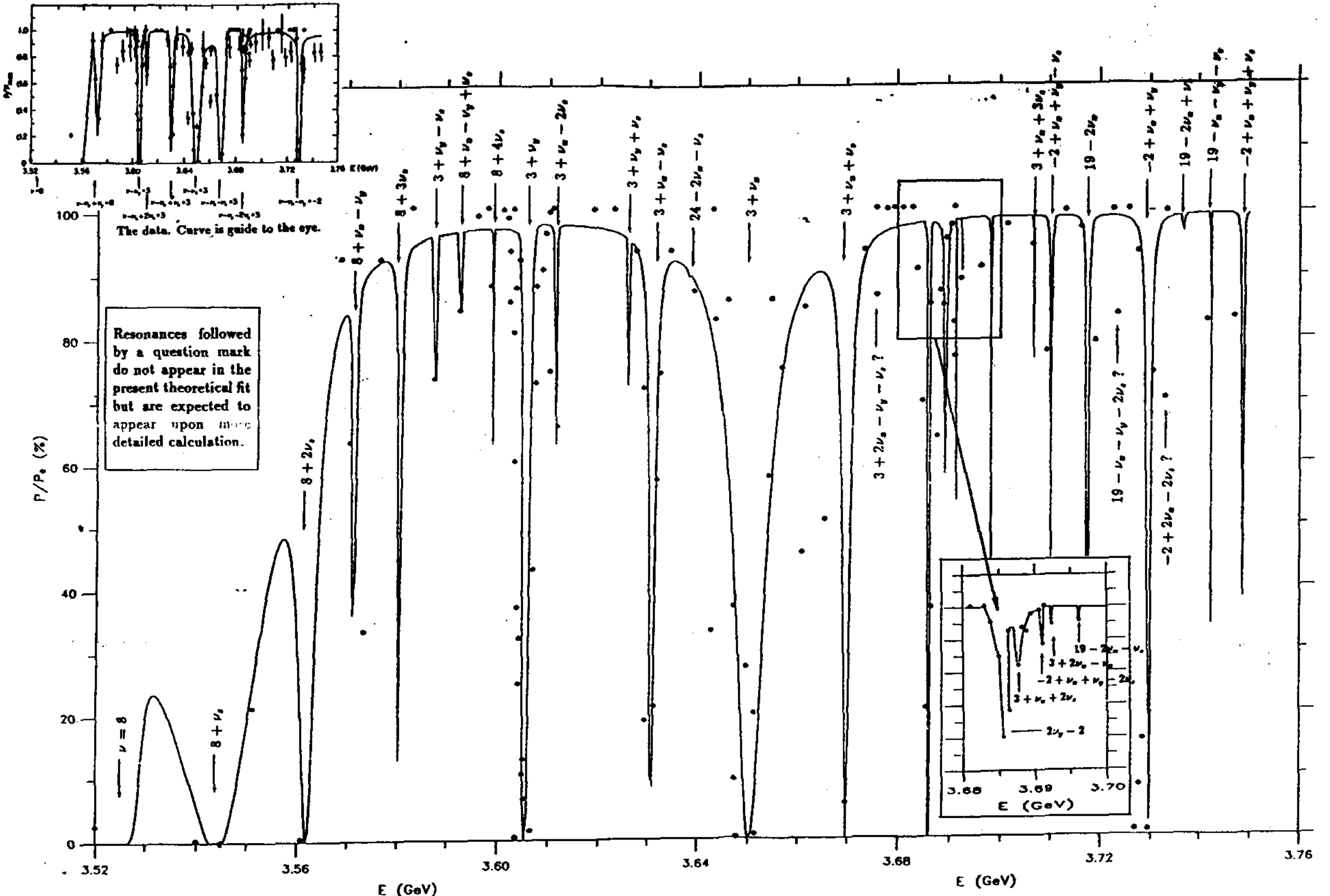
Fit to Data

A theoretical fit to the SPEAR data [3] is shown in Fig. 1. It was produced using the SMILE program [5]. The details of the fit are given in Ref. [7]. The locations of resonances are given by

$$\nu = k_0 + k_1\nu_x + k_2\nu_y + k_3\nu_s. \tag{5}$$

Fig. 1

THEORETICAL INTERPRETATION OF THE SPEAR POLARIZATION DATA



S.R. Mane, Fermilab, September 1988.

Here ν is the spin tune and ν_i, $i = x, y, s$, are the orbital tunes. The k_i are integers, including zero. Various resonances are identified by the experimenters in the inset to Fig. 1 [3]. The tunes were read off from it, and used in the theortical fit. In fact two sets of tunes were used, above and below 3.60 GeV, because the tunes were not constant throughout the whole of Fig. 1. Another important fact to note is that the resonance widths depend on imperfections in the machine, so a set of distorted closed orbits was produced by using different random seeds, and the sum total of these results was used to prepare the fit. This is an important reason why prediction of absolute resonance widths is very difficult in general. A simple closed orbit smoothing procedure was used, which consisted of suppressing the Fourier harmonics of the closed orbit closest to the vertical betatron tune ($\nu_y \simeq 5.18$). The global scale of the random kicks was choen so as to approximately fit the resonance $\nu = 3 + \nu_s$ at 3.65 GeV. The resulting r.m.s. closed orbit amplitude varied from 0.6 to 1.2 mm. In addition, the r.m.s. vertical betatron tunespread was read off from another graph in Ref. [3]. This was attributed to transverse nonlinear tunespread, and a simple model of tunespread was introduced to fit the nonlinear resonances $\nu = 3 + \nu_y$ at 3.605 GeV and $\nu = 2\nu_y - 2$ at 3.686 GeV. After the machine parameters had been fixed in this way, all the resonances were calculated without further modifications.

Although a lot of the input for the theoretical fit involved guesswork, because after nearly ten years detailed information was difficult to find, it is still possible explain some of the puzzling features in the experimental data. Note that the experimental curve in Fig. 1 is a guide to the eye, not a theoretical fit. Some depolarizing resonances are identified. The analysis in Fig. 1 reveals, in addition, the existence of several more narrow resonances in the data, showing that the experimental measurements were very precise. It appears that data ignored as statistical noise experimentally actually were real physics. In particular, one theoretical prediction is that the resonances $\nu = 3+\nu_x \pm 2\nu_s$, at 3.61 GeV and 3.69 GeV, should have approximately equal width. Here ν is the spin tune and ν_i, $i = x, y, s$, are the orbital tunes. It can be clearly seen that the two resonances above have very unequal widths. By using SMILE, I am able to show that the data around 3.69 GeV, drawn as one wide resonance in the inset to Fig. 1, in fact consists of several narrow, nearly overlapping resonances. The resonances $\nu = 3 + \nu_x \pm 2\nu_s$ do in fact appear to have approximately equal width, after the theoretical analysis. A more detailed fit to the data in the vicinity of 3.69 GeV is shown in Fig. 2.

Further details are given in Ref. [7]. In addition, the above reference also contains some theoretical predictions about the behavior of the resonance widths, which should be experimentally testable.

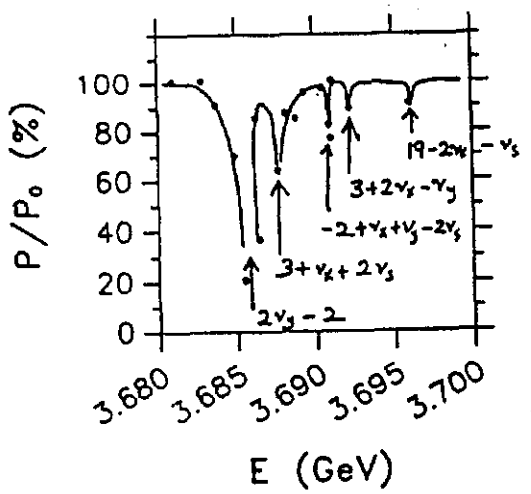


Fig. 2 Resonance spectrum in the region around 3.69 GeV.

Acknowledgements

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